Wavelet Leader Multifractal Analysis for Heart Rate Variability

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Heart Rate Variability Analysis. Analyzing and characterizing Heart Rate Variability (HRV) constitutes a significant public health stake — both for understanding mechanisms regulating heart rate in healthy subjects and for detection and classification of pathologies — that received, and continues to arouse, significant research efforts. After the seminal work in [1], spectral analysis has been massively used to investigate HRV time dynamics. Essentially, it relies on splitting the spectrum into frequency bands and on measuring the respective amounts of energy within each band. To go beyond spectrum analysis, that assumes stationarity and focuses on the sole 2nd order statistical order (the correlation function), numerous efforts were devoted to non stationary, non Gaussian and non linear analyses of HRV, using tools such as time-frequency/scale representations, chaos, entropy, complexity,…(cf. e.g., [2], [3], [4], [5]).

Scale Invariance and Multifractal analysis. An important finding in HRV analysis consists of the observation that, at low frequencies, i.e., essentially for \( f \leq 0.1 \) Hz, HRV time dynamics show no preferred scales: No frequency plays a specific role. It has aroused the use of concepts such as scale invariance, scaling, self-similarity or (multi-)fractal to describe HRV fluctuations. Scale invariance is reconciling the spectrum and time variability analysis perspectives (a healthy subject must show a variability at a chosen scale \( a \), \( V_a(t) = \sup_{u,v \in [t - a/2, t+a/2]} \{ |X(u) - X(v)| \} \) larger than a chosen threshold) and extends them in three key respects: First, the quantity used to analyze HRV needs to be restricted neither oscillation \( |X(u) - X(v)| \), nor to the spectrum, but can be any multiscale quantity \( T_X(a, t) \) depending jointly on the time position \( t \) and on the analysis scale \( a \), the modulus of wavelet coefficients being natural candidates; Second, interest is no longer in the values taken by variability \( V_a(t) \) at a given time, or by Spectrum at a given frequency, but instead in the power law shaped dependence of these values with respect to the analysis scale \( a \) or frequency \( f \); Third, analysis is not restricted to first or second order statistics of data only but aims at probing their whole statistical properties.

In a multiscale (wavelet) framework, these three extensions are efficiently summarized as \( \sum T_X^a(a, t) \simeq a^{\zeta(q)} \), across a large range of scales \( \alpha_m \leq a \leq \alpha_M \), \( \alpha_M/\alpha_m \gg 1 \) and for a range of statistical orders \( q \). The scaling exponents \( \zeta(q) \) are used for HRV characterization or pathology detection. Because the \( \zeta(q) \) can be theoretically related to the fluctuations along time of the local (Hölder) regularity of sample path and to the multifractal spectrum of data, this power-law modeling is often referred to as multifractal analysis, a tool that as already been largely explored in HRV analysis, cf. e.g., [6], [7], [8], [9], [10] and references therein.

Wavelet-Leader based Multifractal analysis. It has recently been shown that a theoretically well-grounded and practically efficient formulation of multifractal analysis (multifractal formalism) should be based on wavelet Leaders, a variation on wavelet coefficients consisting replacing in the modulus of the wavelet coefficient \( |T_X(a, t)| \) located at time \( t \) and scale \( a \), by the supremum of all \( |T_X(a', t')| \) located in a time neighborhood \( |t' - t| \leq a \) and at any finer scale \( a' \leq a \). This formalism enables to test whether data are truly multifractal or better modeled by self-similar processes [11], to estimate efficiently the Hurst parameter \( \tilde{H} = \zeta(2)/2 \) as well as the intermittency parameter, and to show that \( \zeta(2) \) and \( H \) can be tightly related to the LF/HF ratio, commonly used in spectral based HRV analysis [12]. It has notably been successfully applied to the detection of acidosis in intra partum fetal HRV, with emphasis on the decrease of False Positive [13].

References